

Math 43 Quiz Midterm 3 Review Answers

In addition to the following review questions, you must be able to solve any of the questions from the 3D Lines & Planes handout.

- [1] [a] $\langle -2, -4 \rangle \cdot \langle x, y \rangle = 0 \Rightarrow -2x - 4y = 0$
 Let $x = 2$ and $y = -1$
 $\frac{1}{\|\langle 2, -1 \rangle\|} \langle 2, -1 \rangle = \frac{1}{\sqrt{5}} \langle 2, -1 \rangle = \langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \rangle$
- [b] $\|\vec{m}\| = 2\sqrt{5} \Rightarrow \cos \theta = \frac{-2}{2\sqrt{5}} = -\frac{1}{\sqrt{5}}$ and $\sin \theta = \frac{-4}{2\sqrt{5}} = -\frac{2}{\sqrt{5}} \Rightarrow \theta$ in quadrant 3
 $\theta_{REF} = \cos^{-1} \frac{1}{\sqrt{5}} \approx 1.107 \Rightarrow \theta \approx \pi + 1.107 \approx 4.25$
- [c] $2 \langle 8 \cos \frac{2\pi}{3}, 8 \sin \frac{2\pi}{3} \rangle - \langle -2, -4 \rangle = 2 \langle -4, -4\sqrt{3} \rangle - \langle -2, -4 \rangle = \langle -8, -8\sqrt{3} \rangle - \langle -2, -4 \rangle$
 $= \langle -6, 4 - 8\sqrt{3} \rangle = -6\vec{i} + (4 - 8\sqrt{3})\vec{j}$
- [2] [a] $\cos^{-1} \frac{\langle 0, 2, -3 \rangle \cdot \langle -1, -3, 4 \rangle}{\|\langle 0, 2, -3 \rangle\| \|\langle -1, -3, 4 \rangle\|} = \cos^{-1} \frac{-18}{\sqrt{13}\sqrt{26}} \approx 2.94$
- [b] $\langle 0, 2, -3 \rangle \times \langle -1, -3, 4 \rangle = \langle -1, 3, 2 \rangle$
 $\frac{1}{\|\langle -1, 3, 2 \rangle\|} \langle -1, 3, 2 \rangle = \frac{1}{\sqrt{14}} \langle -1, 3, 2 \rangle = \langle -\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}} \rangle$
- [c] $proj_{\langle -1, -3, 4 \rangle} \langle 0, 2, -3 \rangle = \frac{\langle 0, 2, -3 \rangle \cdot \langle -1, -3, 4 \rangle}{\langle -1, -3, 4 \rangle \cdot \langle -1, -3, 4 \rangle} \langle -1, -3, 4 \rangle = \frac{-18}{26} \langle -1, -3, 4 \rangle$
 $= -\frac{9}{13} \langle -1, -3, 4 \rangle = \langle \frac{9}{13}, \frac{27}{13}, -\frac{36}{13} \rangle$
 $\langle 0, 2, -3 \rangle - \langle \frac{9}{13}, \frac{27}{13}, -\frac{36}{13} \rangle = \langle -\frac{9}{13}, -\frac{1}{13}, -\frac{3}{13} \rangle$
 $\langle 0, 2, -3 \rangle = \langle \frac{9}{13}, \frac{27}{13}, -\frac{36}{13} \rangle + \langle -\frac{9}{13}, -\frac{1}{13}, -\frac{3}{13} \rangle$
- [d] $\langle -7 - x, 4 - y, -8 - z \rangle = \langle -1, -3, 4 \rangle$
 $-7 - x = -1, 4 - y = -3, -8 - z = 4 \Rightarrow (x, y, z) = (-6, 7, -12)$
- [e] $\langle a, b, -5 \rangle = m \langle -1, -3, 4 \rangle \Rightarrow a = -m, b = -3m, -5 = 4m \Rightarrow m = -\frac{5}{4}, a = \frac{5}{4}, b = \frac{15}{4}$
- [f] $\langle 7, c, -5 \rangle \cdot \langle -1, -3, 4 \rangle = 0 \Rightarrow -7 - 3c - 20 = 0 \Rightarrow c = -9$
- [3] [a] octant $2 + 4 = 6$
- [b] $(-5 - 4, -2 + 6, 3 - 2) = (-9, 4, 1)$
- [c] $\langle 3 - -3, 2 - 4, -1 - -2 \rangle = \langle 6, -2, 1 \rangle$
- [d] $\langle -3 - -5, 4 - -2, -2 - 3 \rangle = \langle 2, 6, -5 \rangle = 2\vec{i} + 6\vec{j} - 5\vec{k}$
- [e] $\sqrt{4 + 36 + 25} = \sqrt{65}$
- [f] $-\frac{1}{\sqrt{65}} \langle 2, 6, -5 \rangle = \langle -\frac{2}{\sqrt{65}}, -\frac{6}{\sqrt{65}}, \frac{5}{\sqrt{65}} \rangle$
- [g] $\frac{6}{\sqrt{41}} \langle 6, -2, 1 \rangle = \langle \frac{36}{\sqrt{41}}, -\frac{12}{\sqrt{41}}, \frac{6}{\sqrt{41}} \rangle$
- [h] $(\sqrt{41})(3) \cos 2 \approx -7.99$
- [i] $(\sqrt{41})(3) \sin 2 \approx 17.47$
- [j] $\vec{RQ} \times \vec{RP} = \langle 6, -2, 1 \rangle \times \langle -2, -6, 5 \rangle = \langle -4, -32, -40 \rangle = -4 \langle 1, 8, 10 \rangle$
 $\frac{1}{2} \|\langle -4 \langle 1, 8, 10 \rangle \rangle\| = 2 \|\langle 1, 8, 10 \rangle\| = 2\sqrt{165}$
- [k] $\cos^{-1} \frac{\langle 6, -2, 1 \rangle \cdot \langle -2, -6, 5 \rangle}{\|\langle 6, -2, 1 \rangle\| \|\langle -2, -6, 5 \rangle\|} = \cos^{-1} \frac{5}{\sqrt{41}\sqrt{65}} \approx 1.47 \text{ radians}$
- [l] $\langle 4, 0, -5 \rangle \cdot \langle -5 - 3, -2 - 2, 3 - -1 \rangle = \langle 4, 0, -5 \rangle \cdot \langle -8, -4, 4 \rangle = -52$
- [m] normal vector $= \vec{RQ} \times \vec{RP} = \langle -4, -32, -40 \rangle$ or $\langle 1, 8, 10 \rangle$
 $1(x + 5) + 8(y + 2) + 10(z - 3) = 0 \Rightarrow x + 8y + 10z - 9 = 0$

[n] $x = -5 + 6t, \quad y = -2 - 2t, \quad z = 3 + t$

[o] direction vector = $\langle -2, -3, 1 \rangle$ or $\langle 2, 3, -1 \rangle$

$$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z+1}{-1} \text{ or } -z-1$$

[p] center = $(\frac{-5+3}{2}, \frac{-2+2}{2}, \frac{3+1}{2}) = (-1, 0, 1)$

$$\text{radius}^2 = (3 - (-1))^2 + (2 - 0)^2 + (-1 - 1)^2 = 24$$

$$(x+1)^2 + y^2 + (z-1)^2 = 24$$

[4] $x < 0, y > 0, z > 0 \Rightarrow$ octant 2

$x < 0, y < 0, z > 0 \Rightarrow$ octant 3

$x > 0, y > 0, z < 0 \Rightarrow$ octant 1 + 4 = 5

$x > 0, y < 0, z < 0 \Rightarrow$ octant 4 + 4 = 8

[5] [a] $x^2 - 4x + y^2 + 6y + z^2 + 10z = -29$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) + (z^2 + 10z + 25) = -29 + 4 + 9 + 25$$

$$(x-2)^2 + (y+3)^2 + (z+5)^2 = 9$$

center = $(2, -3, -5)$

radius = 3

[b] $(x-2)^2 + (y+3)^2 + (0+5)^2 = 9 \Rightarrow (x-2)^2 + (y+3)^2 = -16$

no xy -trace

$$(x-2)^2 + (0+3)^2 + (z+5)^2 = 9 \Rightarrow (x-2)^2 + (z+5)^2 = 0$$

xz -trace is point $(2, 0, -5)$

$$(0-2)^2 + (y+3)^2 + (z+5)^2 = 9 \Rightarrow (y+3)^2 + (z+5)^2 = 5$$

yz -trace is circle in yz -plane, center = $(0, -3, -5)$, radius = $\sqrt{5}$